An Accurate Broadband Measurement of Substrate Dielectric Constant

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***Abstract*—An improved two-microstrip line method is proposed for a simple and accurate measurement of the dielectric constants of substrates. The errors due to the transition mismatches and connection repeatability are removed by using the selection algorithm of the best data set in multiple measurements, based on the minimum error cost concept. The measurement data for CGP-500 substrate in broad frequency range (0.5–25.5 GHz) are shown and the result agrees quite well with the theoretical one.**

I. Introduction

A broadband measurement of a dielectric constant of a microwave material can be made by using the transmission/reflection method developed by Weir [1]. For general transmission lines, Enders [2] has shown a method for determining all the properties of an unknown line and their junctions to the line using three different lengths of the unknown line. On the other hand, Das *et al.* [3] have developed a two-line method to measure substrate permittivities. Although the method is simple, quick, and reliable to use, it has several drawbacks. One is that the technique works well on the condition that the transition effect of coax-to-microstrip is relatively small. This means that the approximate substrate permittivity should be known before the measurement, so that the characteristic impedance of the test section can be designed in the vicinity of 50 Ω. The other is that the method gives us an accurate result only if the electrical length of lines is long. In this letter, we present an improved two-microstrip line method in which a wide-band measurement is possible with short lines. Also, the transition effects can be removed systematically without *a priori* knowledge of the substrate dielectric constant.

II. Theory

A. Improved Two-Microstrip-Line Method

Fig. 1 shows two microstrip lines with same characteristic impedance (unnecessary to be 50 Ω). One is longer than the other. For both line (a) and (b), the measured two-port parameters expressed in **ABCD** matrix form can be considered as a product of three parts: an input matrix $\mathbf{X}$ including the input coax-to-microstrip transition, transmission line $\mathbf{T}$, and an output matrix $\mathbf{Y}$, including the output coax-to-microstrip transition

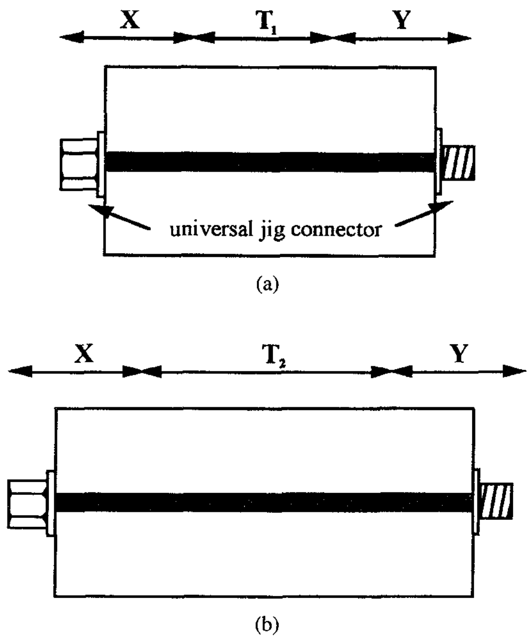

Fig. 1. Two microstrip lines with same characteristic impedance and definition of $\mathbf{X}$, $\mathbf{T}_i$, and $\mathbf{Y}$. (a) Shorter line. (b) Longer line.

$$\mathbf{M}_1 = \mathbf{X}\mathbf{T}_1\mathbf{Y} \tag{1}$$

$$\mathbf{M}_2 = \mathbf{X}\mathbf{T}_2\mathbf{Y} \tag{2}$$

where $\mathbf{M}_i$, $\mathbf{X}$, $\mathbf{T}_i$, and $\mathbf{Y}$ are $\mathbf{ABCD}$ matrices for the corresponding sections as in the figure. Multiplying the matrix $\mathbf{M}_1$ by the inverse matrix of $\mathbf{M}_2$, we obtain (3)

$$\mathbf{M}_1\mathbf{M}_2^{-1} = \mathbf{X}\mathbf{T}_1\mathbf{T}_2^{-1}\mathbf{X}^{-1}. \tag{3}$$

In (3), notice that $\mathbf{M}_1\mathbf{M}_2^{-1}$ is the similar transformation of $\mathbf{T}_1\mathbf{T}_2^{-1}$. Using the fact that the trace, which is defined as the sum of the diagonal elements, does not change under the similar transformation in the matrix calculation, we can deduce (4)

$$\mathrm{Tr}(\mathbf{M}_1\mathbf{M}_2^{-1}) = \mathrm{Tr}(\mathbf{T}_1\mathbf{T}_2^{-1}) = 2\cosh(\gamma\Delta d) \tag{4}$$

where γ is the complex propagation constant of the microstrip line and Δd is the length difference of two microstrip lines.

The complex effective dielectric constant $\epsilon_{\text{eff}} = \epsilon'_{\text{eff}} - j\epsilon''_{\text{eff}}$ of the substrate is found from (4)

$$\epsilon_{\text{eff}} = \left[\frac{1}{\beta_0\Delta d}\cosh^{-1}\left\{\frac{1}{2}\mathrm{Tr}(\mathbf{M}_1\mathbf{M}_2^{-1})\right\}\right]^2 \tag{5}$$

where β_0 is the phase constant in free space.

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B. Minimizing the Error Due to Connector Repeatabilities

Although the theory of the previous two-microstrip-line method is simple and quick, the accuracy becomes poor if the coax-to-microstrip transition is not reproducible for two-line measurements. In this case, (3) is not a similar transformation anymore, and (5) gives us quite erroneous results for ϵ_{eff}. In reality, the connection between the coax and the microstrip line in the universal jig is not repeatable. Therefore, (3) does not guarantee a similar transformation relation between $\mathbf{M}_1\mathbf{M}_2^{-1}$ and $\mathbf{T}_1\mathbf{T}_2^{-1}$. For real measurements of each lines, the $\mathbf{ABCD}$ chain matrix can be expressed by

$$\mathbf{M}_1 = \mathbf{X}\mathbf{T}_1\mathbf{Y} \tag{6}$$
$$\mathbf{M}_2 = \tilde{\mathbf{X}}\mathbf{T}_2\tilde{\mathbf{Y}} \tag{7}$$

where $\mathbf{X}$ and $\tilde{\mathbf{X}}$ ($\mathbf{Y}$ and $\tilde{\mathbf{Y}}$) are the matrices for the input (output) ports of the line (a) and the line (b), respectively. These matrices are assumed to be different because the experimental setup, such as the position of the center pin in the universal jig, can not be reproduced exactly as that of the previous measurement. To obtain a similar transformation of transmission lines from the measured data, we consider four equations

$$\mathbf{M}_1\mathbf{M}_2^{-1} = \mathbf{X}\mathbf{T}_1\mathbf{Y}\tilde{\mathbf{Y}}^{-1}\mathbf{T}_2^{-1}\tilde{\mathbf{X}}^{-1} \tag{8}$$
$$\mathbf{M}_2\mathbf{M}_1^{-1} = \tilde{\mathbf{X}}\mathbf{T}_2\tilde{\mathbf{Y}}\mathbf{Y}^{-1}\mathbf{T}_1^{-1}\mathbf{X}^{-1} \tag{9}$$
$$\mathbf{M}_1^{-1}\mathbf{M}_2 = \mathbf{Y}^{-1}\mathbf{T}_1^{-1}\mathbf{X}^{-1}\tilde{\mathbf{X}}\mathbf{T}_2\tilde{\mathbf{Y}} \tag{10}$$
$$\mathbf{M}_2^{-1}\mathbf{M}_1 = \tilde{\mathbf{Y}}^{-1}\mathbf{T}_2^{-1}\tilde{\mathbf{X}}^{-1}\mathbf{X}\mathbf{T}_1\mathbf{Y}. \tag{11}$$

In general, the trace of any one of (8)–(11) is different from the others because $\mathbf{X}$ and $\tilde{\mathbf{X}}$ ($\mathbf{Y}$ and $\tilde{\mathbf{Y}}$) are assumed to be different. However, the trace of $\mathbf{M}_1\mathbf{M}_2^{-1}$ and $\mathbf{M}_2\mathbf{M}_1^{-1}$ (or the trace of $\mathbf{M}_1^{-1}\mathbf{M}_2$ and $\mathbf{M}_2^{-1}\mathbf{M}_1$) are the same if $\mathbf{X}$ is equal to $\tilde{\mathbf{X}}$ and $\mathbf{Y}$ is equal to $\tilde{\mathbf{Y}}$. Therefore, we obtain two convergence equations given by

$$\mathrm{Tr}(\mathbf{M}_1\mathbf{M}_2^{-1}) - \mathrm{Tr}(\mathbf{M}_2\mathbf{M}_1^{-1}) \Rightarrow 0 \quad \text{as } \mathbf{Y} \to \tilde{\mathbf{Y}}\,|_{\mathbf{X}=\tilde{\mathbf{X}}} \tag{12}$$
$$\mathrm{Tr}(\mathbf{M}_1^{-1}\mathbf{M}_2) - \mathrm{Tr}(\mathbf{M}_2^{-1}\mathbf{M}_1) \Rightarrow 0 \quad \text{as } \mathbf{X} \to \tilde{\mathbf{X}}\,|_{\mathbf{Y}=\tilde{\mathbf{Y}}}\,. \tag{13}$$

Using these convergence equations, we can define the error cost function as

$$e = \frac{1}{n}\sum_{f_{\text{start}}}^{f_{\text{stop}}} \{\|\mathrm{Tr}(\mathbf{M}_1\mathbf{M}_2^{-1}) - \mathrm{Tr}(\mathbf{M}_2\mathbf{M}_1^{-1})\| + \|\mathrm{Tr}(\mathbf{M}_1^{-1}\mathbf{M}_2) - \mathrm{Tr}(\mathbf{M}_2^{-1}\mathbf{M}_1)\|\} \tag{14}$$

where n is the number of the measured frequency points and $\|\cdot\|$ is a suitable norm.

To obtain the minimum error cost, the independent measurements of lines are carried out several times (more than 10 times) and the error cost for each measured data set is calculated. As implied in (12) and (13), the measurement data with minimum error cost correspond to the measurement with minimum transition effects for the input and output coax-to-microstrip lines that will give us the most accurate results for the effective dielectric constant of substrate by (5).

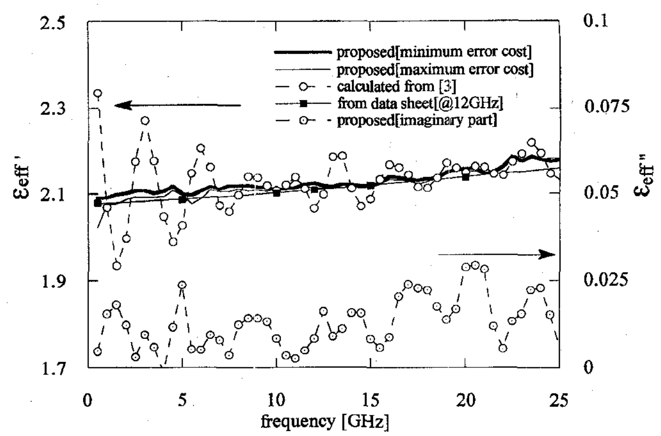

Fig. 2. Measurement data of effective dielectric constant for CGP-500 (width of line = 1 mm, thickness of dielectric = 0.564 mm, thickness of metal = 18 μm).

III. Experimental Results and Discussion

The proposed method is used to measure the effective dielectric constant of CGP-500 (ϵ_r = 2.6 @12 GHz) substrate from Chukoh in broadband range (0.5–25.5 GHz) using a vector network analyzer. Two microstrip lines of 30 mm and 40 mm lengths having 1 mm widths are etched on the substrate of 0.6 mm thickness. To minimize the repeatability error, each line is measured 10 times independently using a universal jig. Then the error cost for the 10 × 10 data sets are calculated.

Fig. 2 shows the measurement data obtained by using the proposed technique. The results are quite stable even in the low-frequency band in which the difference of electrical length of two lines becomes small. Also, we can see the improvement of results obtained by using the selection algorithm compared with the worst case measurement results. The improvement is significant in the low-frequency band.

In order to demonstrate the accuracy of our method, the effective dielectric constant is calculated by the *Linecalc*™ for the same microstrip line using the dielectric constant of 2.6, which is found on the data sheet of CGP-500 at 12 GHz. The results agree quite well and show only 0.33% difference at 12 GHz. We can find out the effective imaginary part of the dielectric constant using (5). The results are shown in Fig. 2. The fluctuations of ϵ''_{eff} caused by imperfect de-embedding of the transitions.

IV. Conclusion

A two-microstrip-line method is proposed for the accurate wideband measurement of a substrate dielectric constant. The proposed technique can be used for microstrip lines with arbitrary width. In this technique, the measurement errors caused by the coax-to-microstrip transitions are minimized by multiple measurements and choosing the measurement data set with minimum error cost. Therefore, the repeatability problem can be systematically reduced. The method does not require the additional calibration kit except the 3.5-mm coaxial kit and gives the improved data even in the low-frequency band, where the difference of electrical length between two lines becomes too small to cause errors. The equations developed in this paper can be applied in measuring the propagation constants of any transmission lines such as striplines, coplanar lines, etc.

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